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06MAT31

Third Semester B.E. Degree Examination, June/July 2013
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain the Fourier series of f(x) defined by
$$f(x)=\begin{cases}-\pi & x\in[-\pi,\ 0]\\ x & x\in[0,\ \pi]\end{cases}$$
(06 Marks)

b. Obtain the half range cosine series and also half range sine series of f(x) defined by $f(x) = lx - x^2\ \ x \in [0, l]$. **(07 Marks)**

c. Express 'y' in a FOuroier series up to first harmonics using the tabular values of x and y given below: **(07 Marks)**

x°	0	30	60	90	120	150	180	210	240	270	300	330
y	1.8	1.1	0.3	0.16	0.5	1.3	2.16	1.25	1.3	1.52	1.76	2.00

2 a. Find the Fourier transform of f(x) defined by
$$f(x)=\begin{cases}1-x^2 & \text{for } |x|\le 1\\ 0 & \text{for } |x|>1\end{cases} \text{ and hence evaluate } \int_0^\infty \frac{\sin x - x\cos x}{x^3}\,dx.$$
(06 Marks)

b. Find the fourier sine transform of $f(x) = e^{-ax}/x \quad (a > 0)$. **(07 Marks)**

c. Solve the integral equation given below:
$$\int_0^\infty f(x)\cos(sx)\,dx = \begin{cases}1-s & 0\le s\le 1\\ 0 & s>1\end{cases},\quad \text{for } f(x).$$
(07 Marks)

3 a. Derive one dimensional heat equation in the form, $\frac{\partial u}{\partial t} = C^2\frac{\partial^2 u}{\partial x^2}$, under suitable assumptions. **(06 Marks)**

b. Obtain D'Alembert's solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = C^2\frac{\partial^2 y}{\partial x^2}$. **(07 Marks)**

c. A string is stretched and fastened to two points 'l' apart. Motion is set by displacing the string in the form $u = a\sin(\pi x/l)$ from which it is released at time t = 0. Show that the displacement at any point at a distance 'x' from one end at time 't' is given by
$$u(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$$
(07 Marks)

4 a. Form the partial differential equation by eliminating the arbitrary functions f and g from the equation $z = f(x) + e^y g(x)$. **(06 Marks)**

b. Solve: $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$ by the method of separation variables. **(07 Marks)**

c. Solve: $(y + zx)\frac{\partial z}{\partial x} - (x + yz)\frac{\partial z}{\partial y} = x^2 - y^2$ **(07 Marks)**

PART – B

5 a. Obtain Newton-Raphson iterative formula to find a positive root of the equation $3x = \cos x + 1$, hence find the root corrected to three decimal places. **(06 Marks)**

b. Solve the following system of equations by Gauss elimination method:
$x + 2y + 2z = 1, \quad 2x + y + z = 2, \quad 3x + 2y + 2z = 3, \quad y + z = 0$ **(07 Marks)**

c. Using power method find the dominant eigen value and the corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$, by taking initial eigen vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Carry out three iterations. **(07 Marks)**

6 a. Applying Newton's forward interpolation formula, find 'y' at x = 160. Given **(06 Marks)**

x :	100	150	200	250	300	350	400
y :	10.63	13.03	15.04	16.81	18.42	19.90	21.27

b. Using Newton's divided difference formula fit an interpolating polynomial for the data given below and hence find 'y' at x = 2. **(07 Marks)**

x :	0	1	4	8	10
y :	-5	-14	-125	-21	355

c. Using Simpson's 3/8th rule, evaluate $\int_4^{5.2} \log_e x \, dx$. (Use seven ordinates) **(07 Marks)**

7 a. With usual notation derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y'}\right) = 0$ **(06 Marks)**

b. Find the curve on which the functional $\int_0^{\pi/2} [(y')^2 - y^2 + 2xy] \, dx$, with $y(0) = y(\pi/2) = 0$ can be extremized. **(07 Marks)**

c. Define "geodesic" and show that it is a straight line on a plane surface. **(07 Marks)**

8 a. Find the z-transforms of $\cos(n\theta)$ and $\sin(n\theta)$. **(06 Marks)**

b. Find the inverse z-transform of $\frac{20z^2 + z}{(z-2)(z-3)}$. **(07 Marks)**

c. Solve the difference equation $u_{n+2} - 3u_{n+1} + 2u_n = 0$ where $u_0 = 0$ and $u_1 = -1$ for u_n. **(07 Marks)**

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